Math 241 Sample Problems for Exam 1

Question 1 Don't forget cylindrical and spherical coordinates and quadric surfaces.

Question 2 Let $f(x, y) = x \sin(y/x)$. Find the partial derivatives: $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial y \partial x}$

Question 3 Find and sketch the domain of the function $f(x, y) = \dfrac{5}{\sqrt{10 - 2y^2 - x^2}}$.

Question 4 Evaluate the following limit:

$$\lim_{(x,y) \to (1,2)} \frac{\sqrt{x+y} - \sqrt{3}}{x + y - 3}$$

Question 5 Show that the following limit does not exist as $(x, y) \to (0, 0)$ by considering different paths of approach.

$$\lim_{(x,y) \to (0,0)} \frac{x^2 y + y^2}{x^4 + y^2}$$

Question 6 Let $w = f(u, v)$ be a function whose derivatives of all orders exist. Suppose that $\dfrac{\partial^2 f}{\partial u^2}(0, 2) = 0$, $\dfrac{\partial^2 f}{\partial u^2}(3, 0) = -3$, $\dfrac{\partial^2 f}{\partial u \partial v}(0, 2) = 2$, $\dfrac{\partial^2 f}{\partial u \partial v}(3, 0) = 3$, $\dfrac{\partial^2 f}{\partial v^2}(0, 2) = 1$, $\dfrac{\partial^2 f}{\partial v^2}(3, 0) = -1$. If $u = y + e^{2x}$ and $v = xy$, what is the value of $\dfrac{\partial^2 w}{\partial y^2}$ evaluated at the point $(x, y) = (0, 2)$.

Question 7 Find the direction in which $f(x, y) = x^2 + \cos xy$ increases most rapidly at the point $(1, \pi/2)$. What is the rate at which f changes in that direction? What is the equation of the tangent plane at the point $(1, \pi/2)$?

Question 8 Find the critical points of the function

$$f(x, y) = x^4 - x^2 y + \frac{3}{4} y^2 - 2y + 5$$

and determine all relative maximum, relative minimum, and saddle points.